Measuring Center

Lecture 15
Sections 5.1 - 5.2

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Hampden-Sydney College

Mon, Sep 22, 2008

Outline

Homework Review

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Measuring the Center

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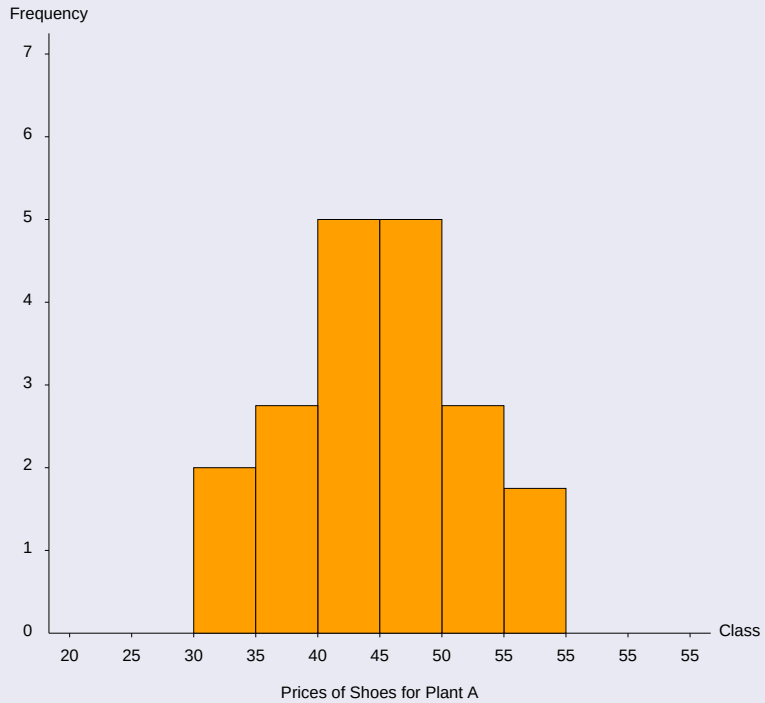
8. Assignment

Homework Review

Exercise 36, p. 262.

Manufacturing Plants A and B produce sports shoes. The histogram of all 20 shoe prices for the 20 different shoes made at Plant A is given by the following figure:



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Exercise 36, p. 262.

(a) How many different shoes produced by Plant A were priced at \$40 or higher?

(b) The following are prices of all 20 sports shoes produced by Plant B:

$$27, 37, 29, 44, 31, 33, 34, 46, 58, 59$$
$$32, 32, 56, 57, 63, 33, 56, 58, 62, 51.$$

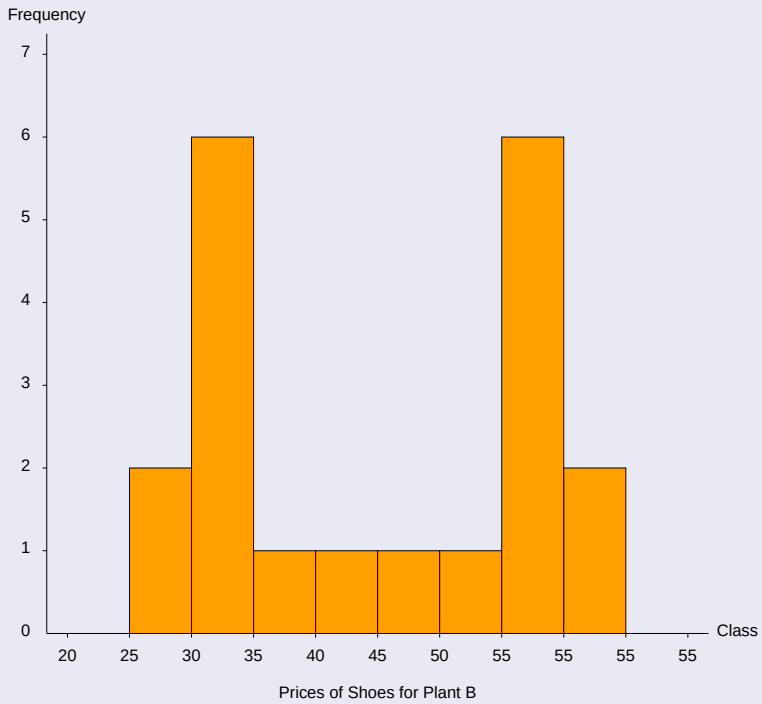
Make a histogram of shoe prices for Plant B. (Use a lower limit $= 20$, an upper limit $= 65$, and a class width $= 5$.)

Homework Review

Solution

(a) 15 different shoes produced by Plant A were priced at \$40 or more.

(b) The histogram for Plant B:



Homework Review

Exercise 36, p. 262.

(c) Select the description(s) appropriate for the distribution of show prices for Plant A.

1. symmetric.
2. unimodal.
3. bimodal.
4. skewed left.
5. skewed right.
6. uniform.

Homework Review

Solution

(c) Select the description(s) appropriate for the distribution of show prices for Plant A.

1. **symmetric**.
2. **unimodal**.
3. bimodal.
4. skewed left.
5. skewed right.
6. uniform.

Homework Review

Exercise 36, p. 262.

(d) Select the description(s) appropriate for the distribution of show prices for Plant B.

1. symmetric.
2. unimodal.
3. bimodal.
4. skewed left.
5. skewed right.
6. uniform.

Homework Review

Solution

(d) Select the description(s) appropriate for the distribution of show prices for Plant B.

1. **symmetric**.
2. unimodal.
3. **bimodal**.
4. skewed left.
5. skewed right.
6. uniform.

Homework Review

Exercise 36, p. 262.

(e) An unknown shoe (that is, it is not known whether it was produced at Plant A or Plant B) has a price tag of \$55. We wish to test the following hypotheses:

H_0: Unknown shoe comes from Plant A.
H_1: Unknown shoe comes from Plant B.

(i) What is the direction of extreme?
(ii) Find the p-value corresponding to the observed price of \$55.
(iii) At $\alpha = 0.05$, what is your decision?

Homework Review

Solution

(e) (i) The direction of extreme is to the left and the right (two-sided).

(ii) Using the Plant A distribution (H_0), there are 2 shoes with prices at least as great as \$55. And there are 2 shoes whose prices are equally extreme to the left ($< \$35$). So the p-value is $4/20 = 0.20$.

(iii) If $\alpha = 0.05$, then we accept H_0 because p-value $> \alpha$.

Measuring the Center

- Often, we would like to have one number that that is “representative" of a population or sample.
- It seems reasonable to choose a number that is near the “center" of the distribution rather than in the left or right extremes.
- But there is no single “correct" way to do this.
- Instead, we will have two (or three) ways to measure the center.

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Definition (Mean)

The mean is the simple average of a set of numbers.

Definition (Median)

The median is the value that divides the set of numbers into a lower half and an upper half.

Definition (Mode)

The mode is the most frequently occurring value in the set of numbers.

Mean, Median, and Mode

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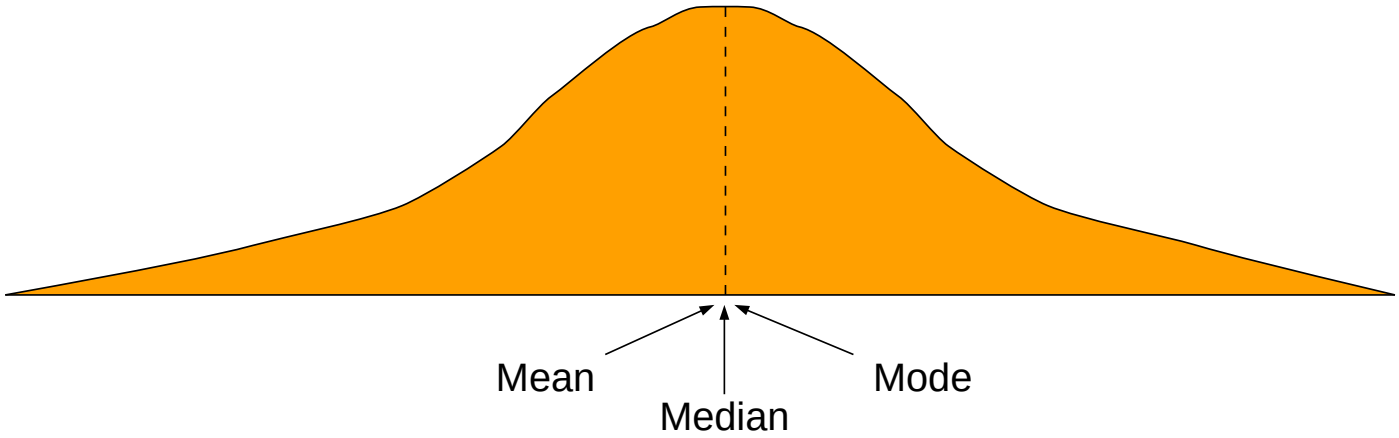
Assignment

- If a distribution is symmetric and unimodal, then the mean, median, and mode are all the same and are all at the center of the distribution.



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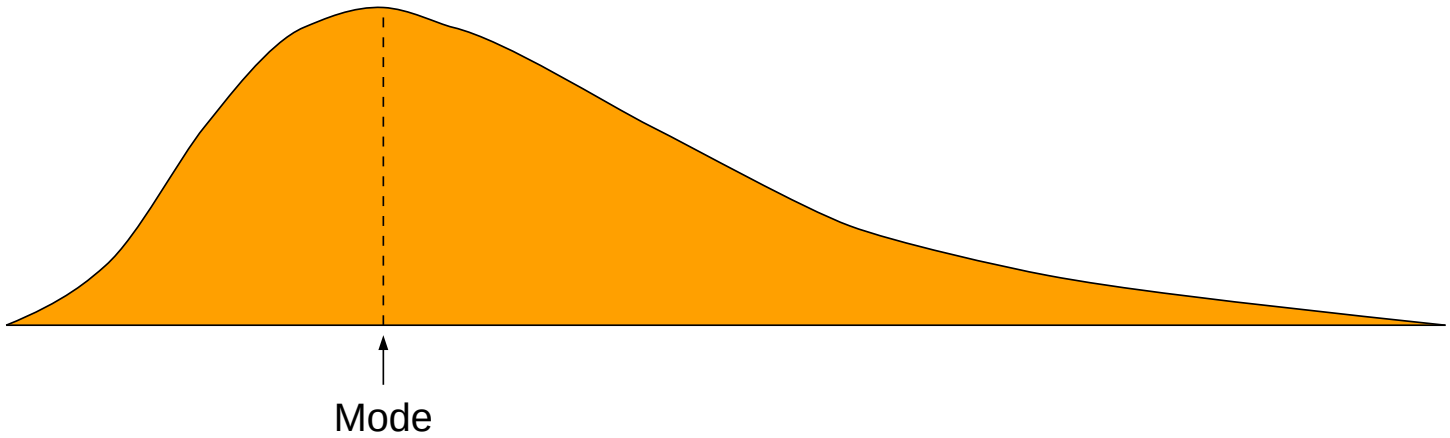
- However, if the distribution is skewed, then the mean, median, and mode are all different.
- The mode is at the peak.



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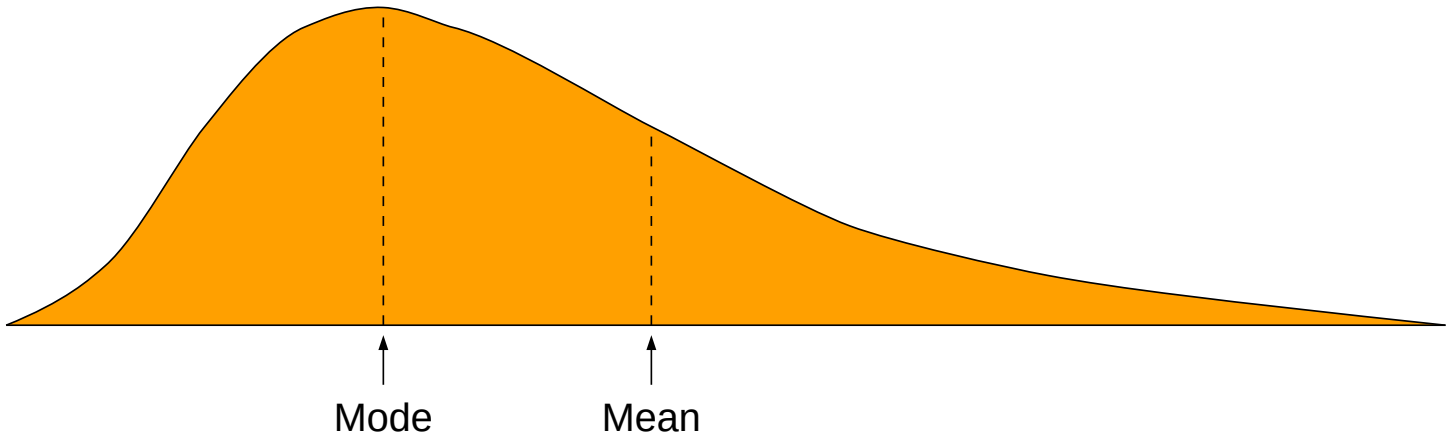
- However, if the distribution is skewed, then the mean, median, and mode are all different.
- The mean is shifted in the direction of skewing.



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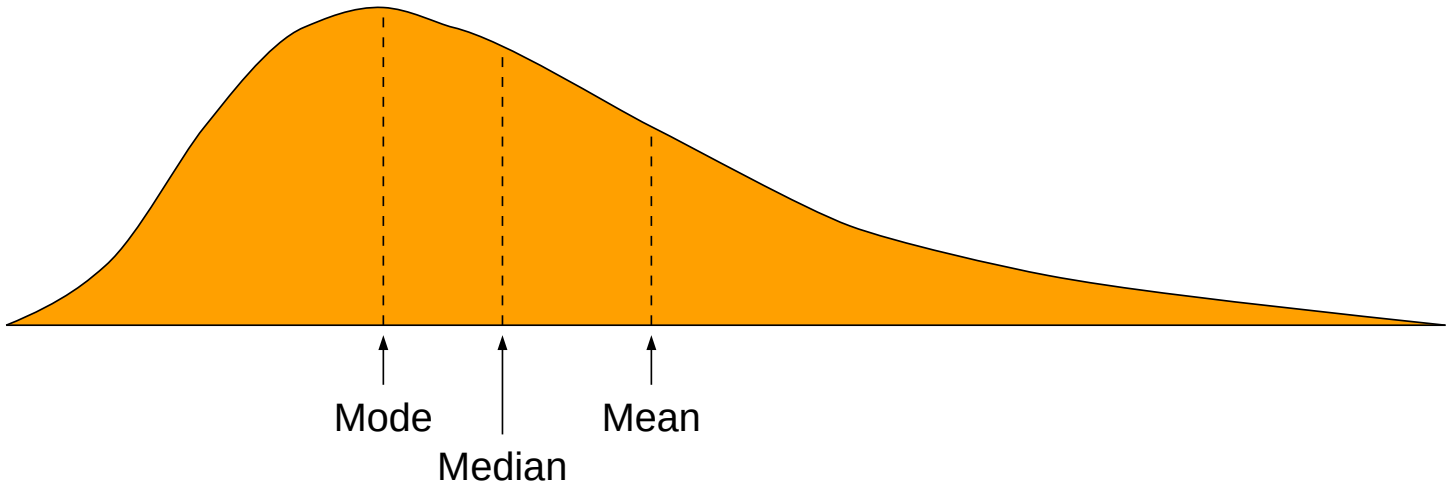
- However, if the distribution is skewed, then the mean, median, and mode are all different.
- The median is between the mode and the mean.



The Median vs. The Mean

- If the data are strongly skewed, then the median generally gives a more representative value.
- If the data are not skewed, then the mean is usually preferred.

The Mean

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- Why is the average usually a good measure of the center?
- If we have only two numbers, the average is half way between them.
- What if we have more than two numbers?
- The mean balances the “deviations” on the left with the “deviations” on the right.

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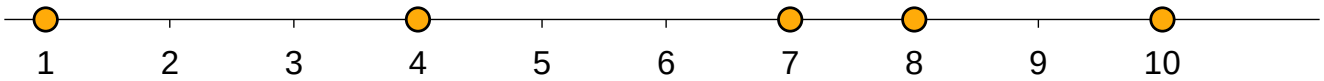

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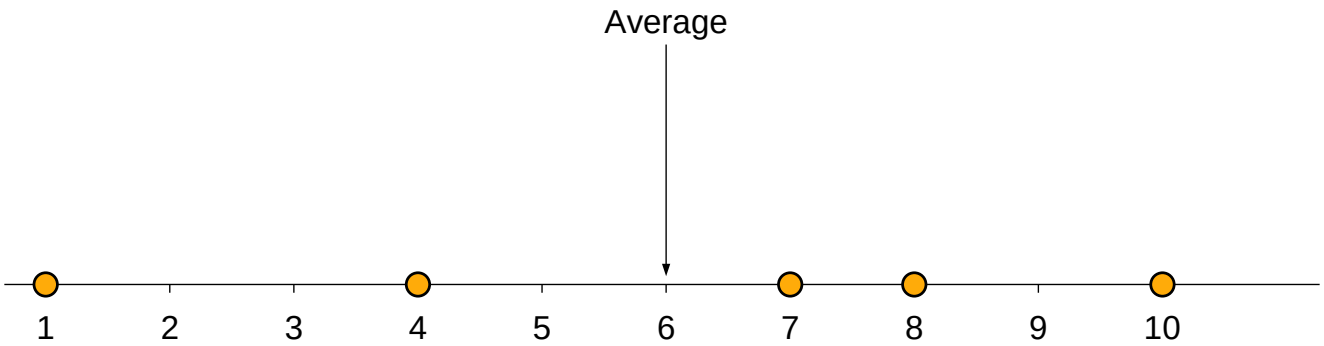

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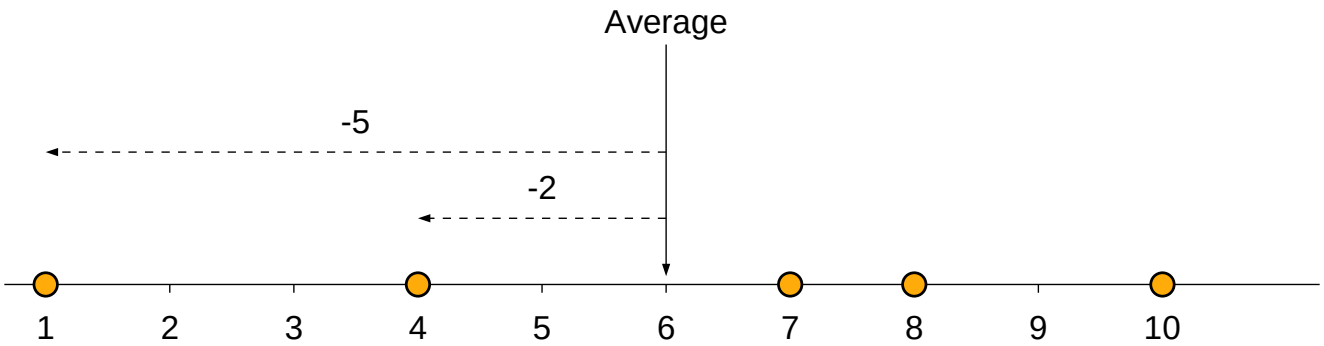

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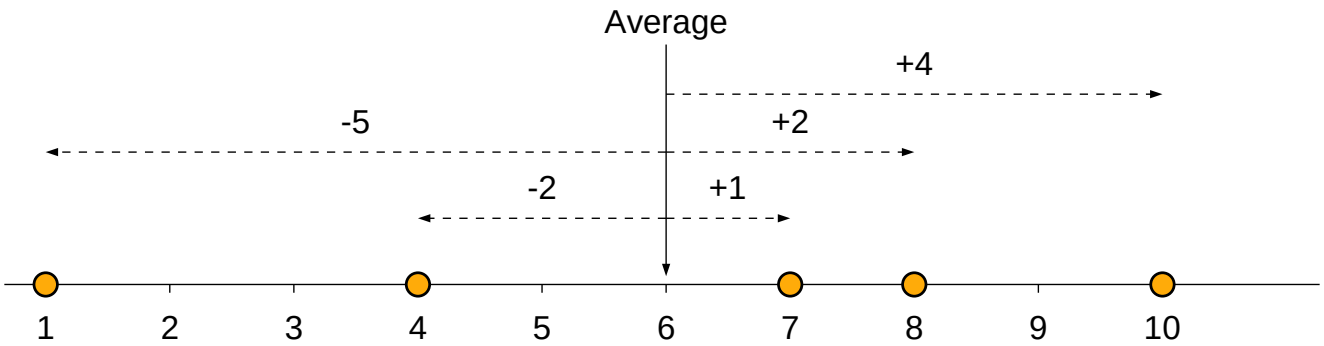

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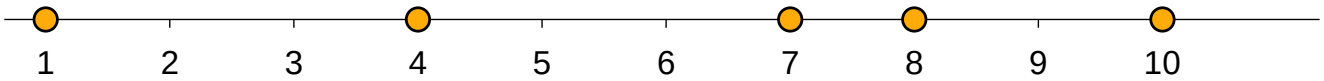

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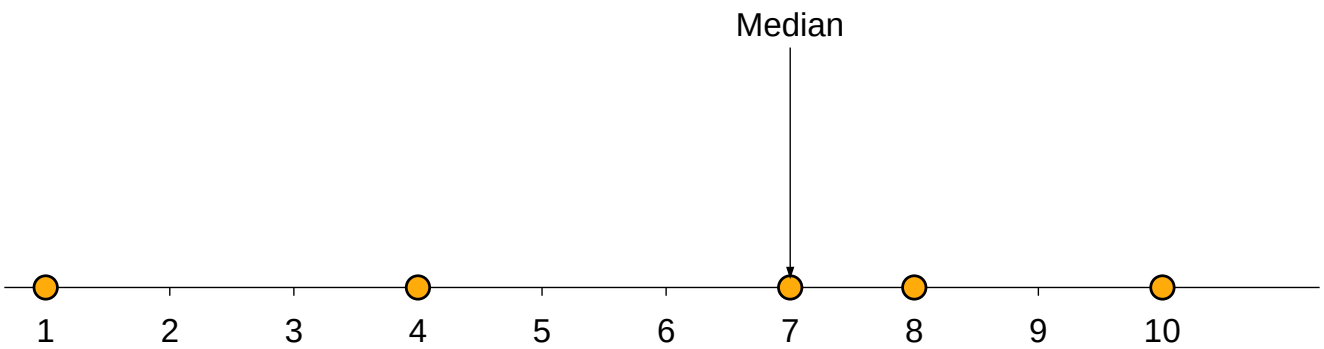

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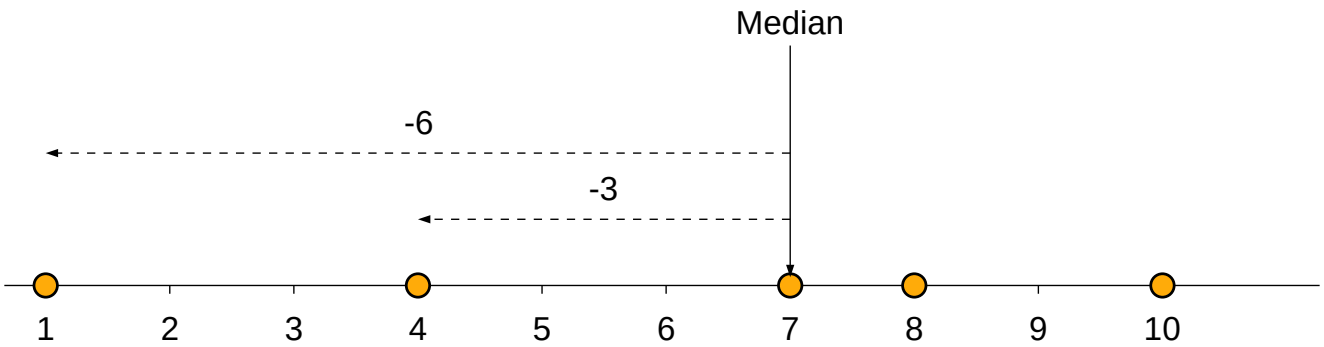

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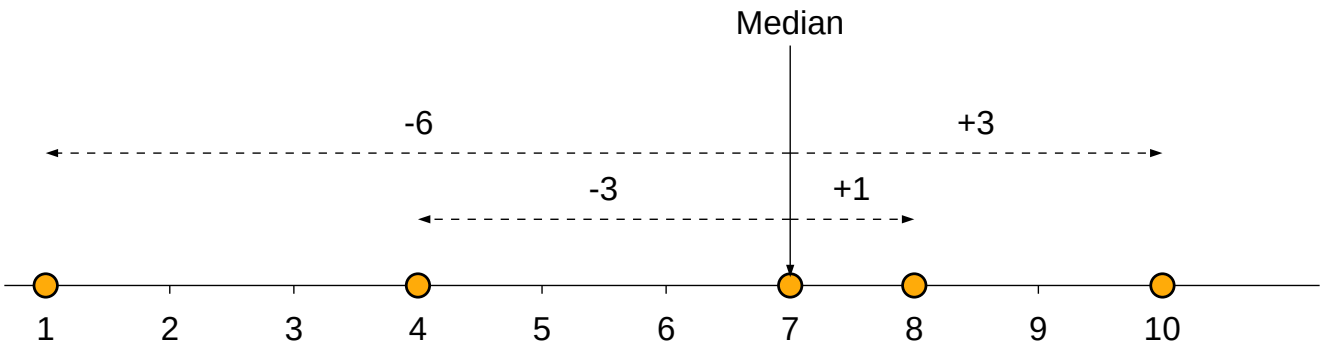

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- We use the letter x to denote a value from the sample or population.
- The symbol Σ means "add them all up."
- So,

$$\sum x$$

means add up all the values in the population or sample (depending on the context).

- Then the sample mean is

$$\frac{\sum x}{n}.$$

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- We denote the mean of a sample by the symbol $\bar{x}$, pronounced "x bar."
- We denote the mean of a population by μ, pronounced "mu" (myoo).
- Therefore,

$$\bar{x} = \frac{\sum x}{n}$$

$$\mu = \frac{\sum x}{N}.$$

TI-83 - The Mean

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TI-83 Means

- Enter the data into a list, say $\texttt{L}_1$.
- Press `STAT > CALC > 1-Var Stats`.
- Press `ENTER`. "`1-Var-Stats`" appears in the display.
- Type $\texttt{L}_1$ and press `ENTER`.
- A list of statistics appears. The first one is the mean.

Practice

Practice

- Rainfall data for Richmond, in centimeters, for 1998 - 2007.

2.82	24.18	0.20	15.60	22.04	7.44
5.16	9.14	37.36	10.19	2.16	17.50

- Find the mean of the data.
- Change the value 37.36 to 75.00. How does that affect the mean?

The Median

Definition (Median)

The median is the value that divides the set of numbers into a lower half and an upper half.

- The median, by definition, is at the 50th percentile.
- It separates the lower 50% of the sample from the upper 50

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- When n is odd, the median is the middle number, which is in position $(n + 1)/2$.
- When n is even, the median is the average of the middle two numbers, which are in positions $n/2$ and $n/2 + 1$.

Practice

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5.16	9.14	37.36	10.19	2.16	17.50

- Find the median of the data.
- Change the value 37.36 to 75.00. How does that affect the median?

TI-83 - The Median

TI-83 Medians

- Follow the same procedure that was used to find the mean.
- When the list of statistics appears, scroll down to the one labeled "`Med`." It is the median.

TI-83 - The Median

Practice

- Use the TI-83 to find the median rainfall.

2.82	24.18	0.20	15.60	22.04	7.44
5.16	9.14	37.36	10.19	2.16	17.50

The Mode

Definition (Mode)

The mode is the most frequently occurring value in the set of numbers.

- The mode is a good indicator of the distribution's central peak, if it has one.

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- The problem is that many distributions do not have a peak or they have several peaks.
- In other words, the mode does not necessarily exist or there may be several modes.

Weighted Means

- The average daily rainfall in Richmond for the period January through July 2008 was 0.136 in/day.
- In August 2008 it was 0.185 in/day.
- What was the average daily rainfall for the period January through August?

Weighted Means

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- It would not be correct to compute the simple average:

$$\begin{aligned} \text{(simple) average} &= \frac{0.136 + 0.185}{2} \\ &= \frac{0.321}{2} \\ &= 0.161 \text{ in/day}. \end{aligned}$$

- Why not?

Weighted Means

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- Instead, compute the weighted average.
- January through July covered
  $31 + 29 + 31 + 30 + 31 + 30 + 31 = 213$ days.
- August covered 31 days.

$$\begin{aligned}
\text{(weighted) average} &= \frac{(213)(0.136) + (31)(0.185)}{213 + 31} \\
&= \frac{28.968 + 5.735}{244} \\
&= \frac{34.703}{244} \\
&= 0.142 \text{ in/day}.
\end{aligned}$$

Practice

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Practice

- A student scores an average of 88 on his first 12 quizzes.
- He scores 62 on his (unlucky) 13th quiz.
- What is his overall average?

Assignment

Homework

- Read Sections 5.1 - 5.2, pages 299 - 311.
- Let's Do It! 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.
- Page 311, exercises 1 - 8.